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## Electric Polarization in a Nanosized, Two-Layer, Ferromagnetic Film with Combined Uniaxial and Cubic Anisotropy in the Layers

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**Abstract**—The average electric polarization arising in a two-layer, nanosized, ferromagnetic film with a combined uniaxial and cubic anisotropy and a vortical distribution of magnetization is studied numerically. Allowance for the cubic anisotropy leads to a multifold increase in the average electric polarization in samples with a positive constant of cubic anisotropy and a significant decrease in samples with a negative constant of cubic anisotropy. Analysis of the hysteresis of the average electric polarization in a magnetic field perpendicular to the film revealed striking differences in the field dependences in films with different cubic anisotropy. If the cubic anisotropy is positive, then the maxima of the average polarization curves shift to the region of low magnetic fields upon an increase in the anisotropy constant. The intensity of the maxima becomes larger, and the hysteresis practically disappears. For films with a negative constant of cubic anisotropy, the maxima of the average polarization curves shift to the region of high fields upon an increase in this constant and the intensity of the maxima becomes significantly smaller.

**Keywords:** two-layer ferromagnetic film, inhomogeneous magnetoelectric effect, electric polarization, hysteresis, magnetic vortex

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#### **INTRODUCTION**

Spintronics-a rapidly developing field of electronics-involves the study of various stable magnetization distributions, which can be used to store and transmit information [1]. These may include skyrmions, magnetic vortices [2], and other magnetic inhomogeneities. The ability to control the state of a magnetic vortexes in different ways, including an electric field in the presence of a magnetoelectric effect [3, 4]. allows them to be considered basic elements for the creation of magnetoelectronic storage devices. The inhomogeneous magnetoelectric effect described by Bar'vakhtar et al. [5] is realized in multiferroics, e.g., ferrite garnets with an inhomogeneous magnetization distribution. This effect gives rise to improper electric polarization, which makes it possible to control such structures via an electric field.

Magnetic vortices can arise in a wide variety of magnetic systems, such as magnetic wires [6]; they can also arise in planar magnetic structures (nanodots [7], nanodiscs [8], and nanoparticles of various shapes) and as part of a more complex magnetic configuration, such as a domain wall with constrictions [9]. A defect with an "easy-axis" anisotropy is created to form a

magnetic vortex in a nanodot with an "easy-plane" anisotropy [10].

Earlier, we considered the process of vortex nucleation and the inhomogeneous magnetoelectric effect in a two-layer, nanosized film with an easy-plane anisotropy and an easy-axis surface anisotropy [11]. For a two-layer exchange-coupled iron-garnet film, the layers of which have an easy-plane and an easyaxis anisotropy, we also studied the conditions for the rise and change of the vortical structure [12], as well as the features of the hysteresis of electric polarization upon magnetization reversal by a magnetic field perpendicular to the film plane [13].

The widely used and comprehensively studied monocrystalline films of garnet and spinel ferrites, in addition to uniaxial anisotropy, also have cubic anisotropy. The objective of this work was to study the magnetization distribution and average electric polarization in two-layer, nanoscale, ferromagnetic films with allowance for the combined cubic and uniaxial anisotropy and to study the effect of the magnitude and sign of cubic anisotropy on the electric polarization upon a reversal of the magnetization of the vortex structure by an external magnetic field perpendicular to the film plane.

#### **1. PROBLEM STATEMENT**

We studied a two-layer film with an upper layer having a uniaxial easy-axis anisotropy and a lower layer with easy-plane anisotropy. The cubic anisotropy is the same for both layers. The film layers have a uniaxial anisotropy of different signs, the same cubic anisotropy, and a finite thickness. The film has finite sizes and a square cross section. The normal to the film coincides with the z axis, the [001] axis of the crystal, and the axis of uniaxial anisotropy. The external magnetic field is parallel to the z axis.

The energy functional of the system has the form

$$W = \sum_{i=1}^{2} \int_{V_{i}} dV \{ E_{\text{cub},i} + E_{\text{u},i} + E_{\text{H},i} + E_{\text{d},i} + E_{\text{ex},i} \} - \int_{S} E_{\text{int}} dS.$$
(1)

It includes the energy of cubic anisotropy  $E_{\text{cub},i} = \frac{K_1}{M_i^4} \{ \mathbf{M}_{x,i}^2 \mathbf{M}_{y,i}^2 + \mathbf{M}_{x,i}^2 \mathbf{M}_{z,i}^2 + \mathbf{M}_{z,i}^2 \mathbf{M}_{y,i}^2 \}$ , the energy of uniaxial magnetic anisotropy  $E_{\text{u},i} = \frac{K_{\text{u},i}}{M_i^2} \mathbf{M}_{z,i}^2$ , the Zeeman energy  $E_{\text{H},i} = -\mathbf{M}_i \mathbf{H}$ , the energy of dipole interaction  $E_{\text{d},i} = -\frac{1}{2} \mathbf{M}_i \mathbf{H}^{(m)}$ , the energy of exchange interaction  $E_{\text{ex},i} = \frac{\alpha_i}{2M_i^2} \left[ \left( \frac{\partial \mathbf{M}_i}{\partial x} \right)^2 + \left( \frac{\partial \mathbf{M}_i}{\partial y} \right)^2 + \left( \frac{\partial \mathbf{M}_i}{\partial z} \right)^2 \right]$ , and the energy of interlayer exchange coupling  $E_{\text{int}} = \frac{J}{M_1 M_2} \mathbf{M}_1 \mathbf{M}_2$ . Here,  $K_1$  is the first constant of cubic anisotropy,  $K_{\text{uv}i}$  are the constants of the uniaxial anisotropy of the layers,  $M_i$  is the saturation magnetiziation of layers,  $\mathbf{H}$  is the external magnetic field,  $\mathbf{H}^{(m)}$  is the magnetic dipole interaction field,  $\alpha_i$  are the exchange coupling constants, and J is the interlayer exchange coupling constant.

#### 2. CALCULATION PROCEDURE

The problem of finding an equilibrium state was solved numerically. The calculations were performed with the OOMMF 3D simulation software package [14] with discretization on a rectangular grid at a step of 5 nm in the *x* and *y* coordinates and 3 nm in the *z* coordinate. The calculation was performed for a two-layer film with the parameters typical of a garnet-ferrite film:  $M_1 \approx 30$  G,  $M_2 \approx 70$  G,  $\alpha \approx 10^7$  erg/cm,  $K_{u, 1} \approx$  $2 \times 10^4$  erg/cm<sup>3</sup>,  $K_{u, 2} \approx -7 \times 10^4$  erg/cm<sup>3</sup>, and J =1 cm<sup>-1</sup>. The cubic anisotropy constants varied from  $-7 \times 10^4$  erg/cm<sup>3</sup> to  $7 \times 10^4$  erg/cm<sup>3</sup>. The studied samples were rectangular nanopillars, the sizes of which varied over a wide range. For illustration, we selected the most typical samples with transverse dimensions of  $200 \times 200$  nm and a thickness of 120 nm.

For a nonuniform magnetization distribution, the magnitude of the electric polarization vector  $\mathbf{P}$  was calculated with the formula [15]

$$\mathbf{P} = \gamma \chi_{e} [(\mathbf{M} \nabla) \mathbf{M} - \mathbf{M} (\nabla \mathbf{M})], \qquad (2)$$

where  $\chi_e$  is the electric polarizability and  $\gamma$  is the coefficient of inhomogeneous magnetoelectric interaction. We will further calculate the polarization vector related to the product of these quantities and the squared saturation magnetization:  $\mathbf{P} \rightarrow \mathbf{P}/\gamma \chi_e M_i^2$ . The dimension of the reduced polarization is cm<sup>-1</sup>. The average polarization is calculated with the formula

$$\langle \mathbf{P} \rangle = \frac{1}{V} \int_{V} \mathbf{P}(x, y, z) dx dy dz.$$
 (3)

In addition, we calculated the polarization averaged over the cell volume,  $\langle P_z \rangle_s$  in a similar manner in order to study the dependence of the polarization on the *z* coordinate. Its transverse dimensions are equal to the transverse dimensions of the film, and the height is equal to 3 nm, which corresponds to the sampling step along the *z* axis.

#### 3. RESULTS AND DISCUSSION

For a vortical magnetization distribution, the components of the average polarization of the sample lying in the film plane are equal to zero, since such a magnetization distribution generates a radial polarization distribution. Therefore, in what follows, we will analyze only the polarization component perpendicular to the film plane,  $\langle P_z \rangle$ . Let us compare the graph showing the change in the average polarization upon the reversal of the magnetization of a two-layer film possessing only uniaxial anisotropy of opposite signs with the graphs showing the change in the average polarization upon the reversal of the magnetization of films possessing, along with a combined uniaxial polarization of different signs, cubic anisotropy. The cubic-anisotropy constant is the same in both layers, but it has positive (Fig. 1) and negative (Fig. 2) values. On the left of the graphs, the constants of cubic anisotropy are indicated. The numbers in the graphs mark the considered points at which the magnetization distribution, as well as the polarization averaged over the cell volume, will be studied further. The magnetization is reversed from the state of saturation along the z axis to the state of saturation against the z axis and, then, vice versa.

It can be seen from Fig. 1 that the maxima of the average polarization curves for positive cubic anisotropy shift to the region of low fields upon an increase in the anisotropy constant. The intensity of the maxima becomes larger, and the local minima located near the zero field also increase; accordingly, the hysteresis practically disappears.





**Fig. 1.** Average polarization  $\langle P_z \rangle$  versus the strength of the external magnetic field. The value of the positive constant of cubic anisotropy is indicated to the left of the curves.

Figure 3a shows the magnetization distribution, and Fig. 4 (curve 1) shows the cell-volume-averaged polarization over the film thickness with only uniaxial anisotropy for a magnetic field  $H_z = 1760$  Oe (point 1 in Fig. 1). It can be seen that the magnetization vortex has not yet formed in this case. The constant of the averaged polarization decreases smoothly between the lower and upper film boundaries. Let us compare it with the magnetization distribution and the cell-volume-averaged polarization for a film with a combined uniaxial and cubic anisotropy of  $4 \times 10^4$  erg/cm<sup>3</sup> for a field  $H_z = 500$  Oe (point 2 in Fig. 1). In this case, the vortex at the lower boundary of the film has already practically formed (Fig. 3b). At the upper boundary, the magnetization is still directed along the z axis. The main contribution to polarization, as seen in Fig. 4, curve 2, comes from the lower layer. The polarization near the lower boundary of the film practically does not change and then gradually decreases.

Let us now consider the change in the polarization and magnetization of a sample in which there is no cubic anisotropy near the minimum of  $\overline{P}_z$  at point 3. With a decrease in the saturating field, the average polarization begins to decrease. In this case, the magnetization vector deviates toward the sample bulk.

When the field reaches -277 Oe (point 3 in Fig. 1), the average polarization is negative. This corresponds to a state in which the magnetization of most of the sublayer with an easy-plane anisotropy has already turned along the field, but the center of the vortex core still contains magnetization that is oriented mainly along the *z* axis (Fig. 3c). In this case, the polarization averaged over the cell volume becomes close to zero in the lower layer and becomes negative in the upper

layer (Fig. 4, curve 7). This leads to a sharp decrease in

the average polarization.

**Fig. 2.** Average polarization  $\langle P_z \rangle$  versus the strength of the

external magnetic field. The value of the negative cubic

anisotropy constant is indicated to the left of the curves.

For a sample with the combined cubic and uniaxial anisotropy near the zero field, the average polarization also has a local minimum in a field of -126 Oe (point 4 in Fig. 1). In this case, the polarization averaged over the layers has a maximum at the center of the sample. Figure 3d shows the corresponding magnetization distribution in the center of the sample and at its boundaries. At the lower boundary of the sample, the magnetization, with the exception of the vortex core, already lies in the (x, y) plane. At the upper boundary, although the vortex structure is already beginning to form, the magnetization is still oriented against the field. In the middle of the sample, where the maximum of polarization is observed (curve 4 in Fig. 4), the magnetization gradually rotates along the field, although magnetization oriented against the direction of the field remains in the vortex core. Then, with a decrease in the field, the magnetization of the upper layer also rotates and the sample magnetization is reversed.

For films possessing cubic anisotropy with a negative constant, on the contrary, the maxima of the average polarization curves shift to the region of high fields upon an increase in the absolute value of the cubic anisotropy constant and the intensity of the maxima decreases (Fig. 2). Upon a decrease in the cubicanisotropy constant, there is also a difference in the behavior of the average polarization near the zero field: first, we observe an increase in the average polarization and then a rapid decrease.

Let us consider the change in the magnetization distribution and the polarization averaged over the cell



**Fig. 3.** Magnetization distribution at the upper and lower film boundaries in different magnetic fields upon magnetization reversal and (d) magnetization distribution between the film layers. The figure correspond to points (a) I, (b) 2, (c) 3, and (d) 4 in Fig. 1.



**Fig. 4.** Electric polarization averaged over the cell volume,  $\langle P_z \rangle_S$  versus the thickness in magnetic fields  $H_z$ : (1) 1760, (2) 500, (4) -126, (7) -277, (3) -1600, (5) 500, and (6) 450 Oe; curves 1, 2, 4, and 7 were calculated after magnetization to saturation along the z axis, and curves 3, 5, and 6 were calculated after magnetization to saturation in the opposite direction.

volume upon a reversal of the film magnetization from the saturation state against the z axis to the saturation state along the z axis. When the average polarization reaches its minimum at point 1 in Fig. 2, which corresponds to a magnetic field of -1600 Oe, the vortical magnetization distribution does not have time to form (Fig. 5a). In this case, the positive polarization in the lower layer and negative in the upper layer turn out to be virtually equal (curve 3 in Fig. 4). Then, already in a positive field, an increase in  $\langle P_z \rangle$  is observed up to a field of 450 Oe (point 2 in Fig. 2). In this case, a vortex forms in both layers of the film, and the magnetization remains directed against the field everywhere, except for the periphery of the lower film layer (Fig. 5b); therefore, the film polarization turns out to be predominantly positive (curve 6 in Fig. 4). Upon a further increase in the magnetic field, a decrease of  $\langle P_z \rangle$ occurs near the field of 500 Oe (point 3 in Fig. 2). This is associated with the direction of the magnetization along the field (Fig. 5c) and partial compensation of the positive polarization of the lower layer and negative upper layer (curve 5 in Fig. 4). The magnetization and polarization distributions at the maximum values



**Fig. 5.** Magnetization distribution at the upper and lower film boundaries in different magnetic fields during magnetization reversal at points (a) 1, (b) 2, (c) 3 in Fig. 2.

of the average polarization are similar to those for films with only uniaxial anisotropy.

### CONCLUSIONS

The magnetization distribution and average electric polarization in two-layer nanosized ferromagnetic

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films with the combined cubic and uniaxial anisotropy were studied numerically.

For the positive cubic anisotropy, the maxima of the average polarization curves shift to the region of low fields upon an increase in the anisotropy constant and the intensity of its maxima becomes larger, because a magnetic vortex has time to form in the lower film layer, whereas the magnetization in both layers near the maximum remains directed along the field in a film with only uniaxial anisotropy.

For a film with the anisotropy of the layers of only uniaxial easy-plane and easy-axis type, the average polarization turns out to be negative at the points of minimal dependence on the external field. This corresponds to the state in which the magnetization of the greater part of the sublayer with the easy-plane anisotropy has already turned along the field, but the magnetization is still present in the center of the vortex core, which is oriented predominantly against the field.

For films with the cubic anisotropy with a negative constant, conversely, the maxima of the average polarization curves shift to the region of high fields upon an increase in the absolute value of the cubic-anisotropy constant and the intensity of the maxima decreases. In this case, the magnetization and polarization distributions at the maximum values of the average polarization are similar to those for films with only uniaxial anisotropy.

Upon a decrease in the cubic anisotropy constant, a difference is also observed in the behavior of the average polarization near the zero field: first, an increase in the average polarization and then a rapid decrease. Upon an increase in the average polarization near the zero field, a vortex forms in both film layers and the magnetization remains directed against the field everywhere, except for the periphery of the lower film layer; therefore, the film polarization turns out to be predominantly positive. Upon a further increase in the magnetic field, there is a decrease in the average polarization, which is associated with the rotation of the magnetization along the field and the partial compensation of the positive polarization of the lower layer and the negative polarization of the upper layer.

Thus, depending on the sign of the cubic anisotropy constant, the character of the field dependences of  $\langle P_z \rangle$  differs significantly: with a positive anisotropy constant, the  $\langle P_z \rangle$  maximum shifts to the region of high fields; the larger the cubic anisotropy constant, the greater is its decrease.

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